International Workshop on

High Energy Density Physics

Beijing, April 19 - 20, 2010

Two lectures by J. Meyer-ter-Vehn :

1. On fast ignition: Selfsimilar isochoric implosions

   reflects my scientific work on Inertial Fusion (1980 -1995)

2. On laser-driven ultra-thin foils as relativistic mirrors

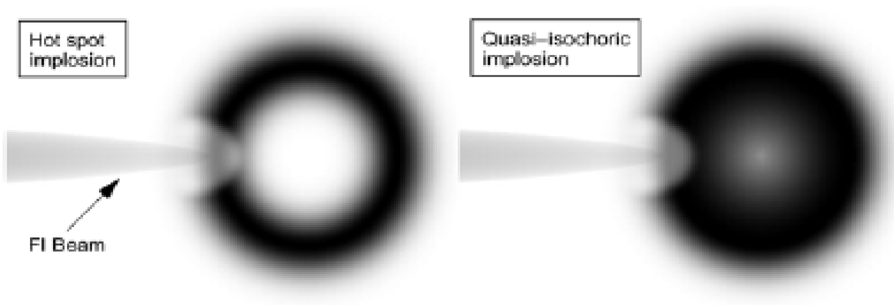
   reflects my work on Relativistic Laser Plasma (1995 - 2010)

On fast ignition:
Selfsimilar isochoric implosions

J. Meyer-ter-Vehn, MPQ Garching

hot spot ignition

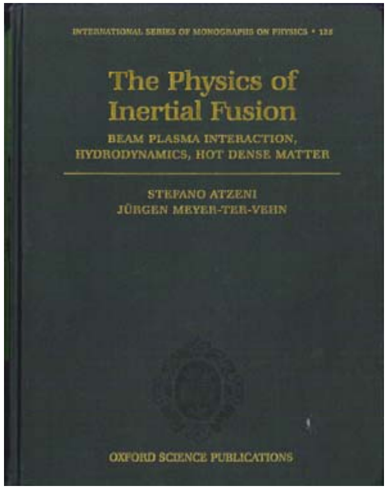
fast ignition



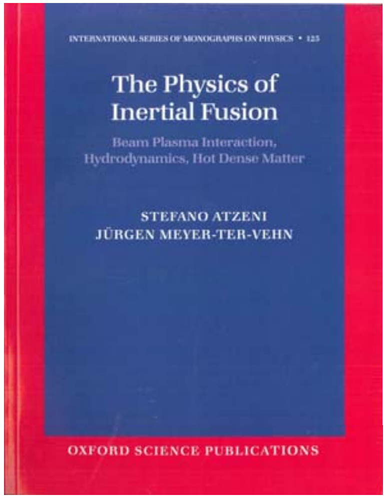
A self-similar isochoric implosion for fast ignition
M.S. Clarke and M.Tabak, Nucl.Fus. 47, 1147 (2007)

The Book

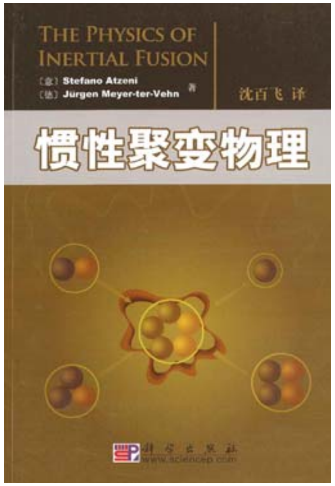
The Physics of Inertial Fusion
S. Atzeni & J. Meyer-ter-Vehn



Oxford University Press 2004



Paperback 2009



Chinese Edition 2009
translated by

Baifei Shen (沈百飞)

Equations of 1D ideal gas dynamics and symmetries

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^{n-1}} \frac{\partial}{\partial r}\left(r^{n-1} \rho u\right) = 0$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial}{\partial r}\left(\frac{\rho c^2}{\gamma}\right) = 0$$

$$\left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial r}\right)\left(\frac{\rho c^2}{\rho^\gamma}\right) = 0$$

Parameters : $n = 1, 2, 3 \quad \gamma = 5/3$

The generators operate on objects

$$\{t, r, u, c, \rho\}$$

Example for stretching :

$$\exp(\varepsilon r \partial_r) F(r) = F(\mathrm{e}^\varepsilon r)$$

time translation $G_1 = \partial_t$

time stretching $G_2 = t\partial_t - u\partial_u - c\partial_c$

space stretching $G_3 = r\partial_r + u\partial_u + c\partial_c - n\rho\partial\rho$

mass stretching $G_4 = \rho\, \partial_\rho$

(time stretching, space stretching, mass stretching: scaling symmetries)

projection $G_5 = t^2\partial_t + rt\partial_r + (r - ut)\partial_u + ct\partial_c - n\rho t\partial\rho$ (for $n = 2/(\rho - 1)$)

satisfied for n=3, γ=5/3

Similarity solutions of gas dynamics

Similarity solutions (Guderley 1942)

velocity $u(r,t) = (\alpha r/t)U(\xi)$

sound velocity $c(r,t) = (\alpha r/t)C(\xi)$

density $\rho(r,t) = \rho_0 \left(r/r_0\right)^{\kappa} G(\xi)$

Similarity coordinate:

$$\xi = \frac{r/r_0}{(t/t_0)^{\alpha}}$$

Two free parameters:

$$\alpha, \kappa,$$

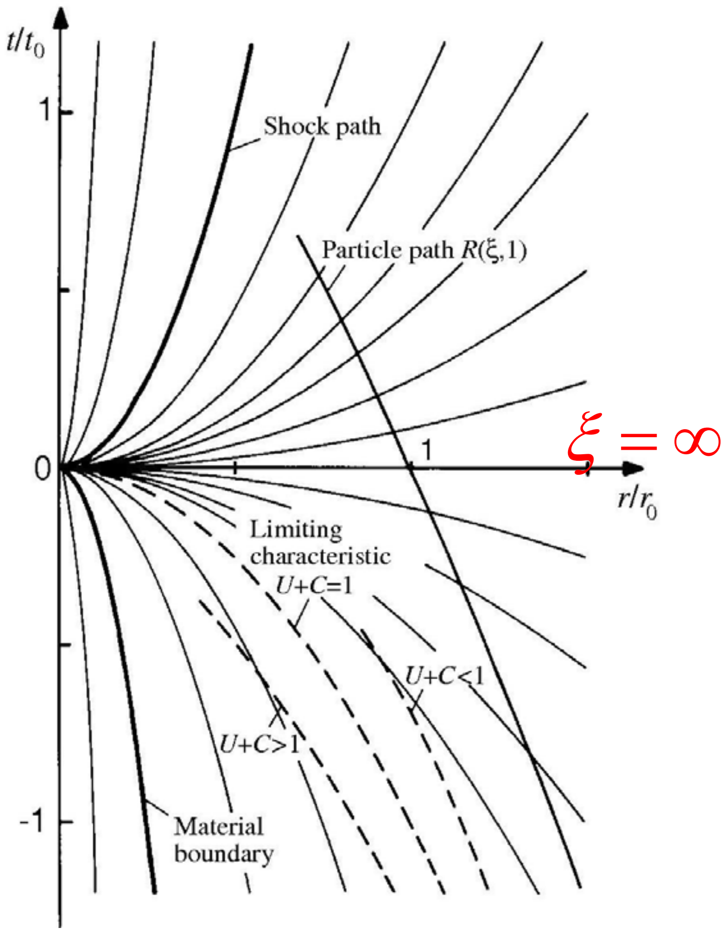
are invatiant under general scaling generator

$$G = t\partial_t + \alpha r\partial_r + (\alpha - 1)u\partial_u + (\alpha - 1)c\partial_c + \alpha\kappa\rho\partial_\rho$$

It allows to reduce the gas dynamics equations to

$$\frac{dU}{dC} = \frac{\Delta_1(U,C)}{\Delta_2(U,C)} \qquad \frac{d\ln\xi}{dC} = \frac{\Delta(U,C)}{\Delta_2(U,C)} \qquad G = G(U,C,\xi)$$

The similarity coordinate $\xi = r / |t|^{\alpha}$



Profiles at $t = 0 \quad (\xi = \infty)$

$$u(r) = u_0 \, r^{\lambda} \qquad (\lambda = 1 - 1/\alpha)$$

$$c(r) = c_0 \, r^{\lambda}$$

$$\rho(r) = \rho_0 \, r^{\kappa}$$

$$p(r) = p_0 \, r^{\kappa - 2\lambda}$$

Entropy

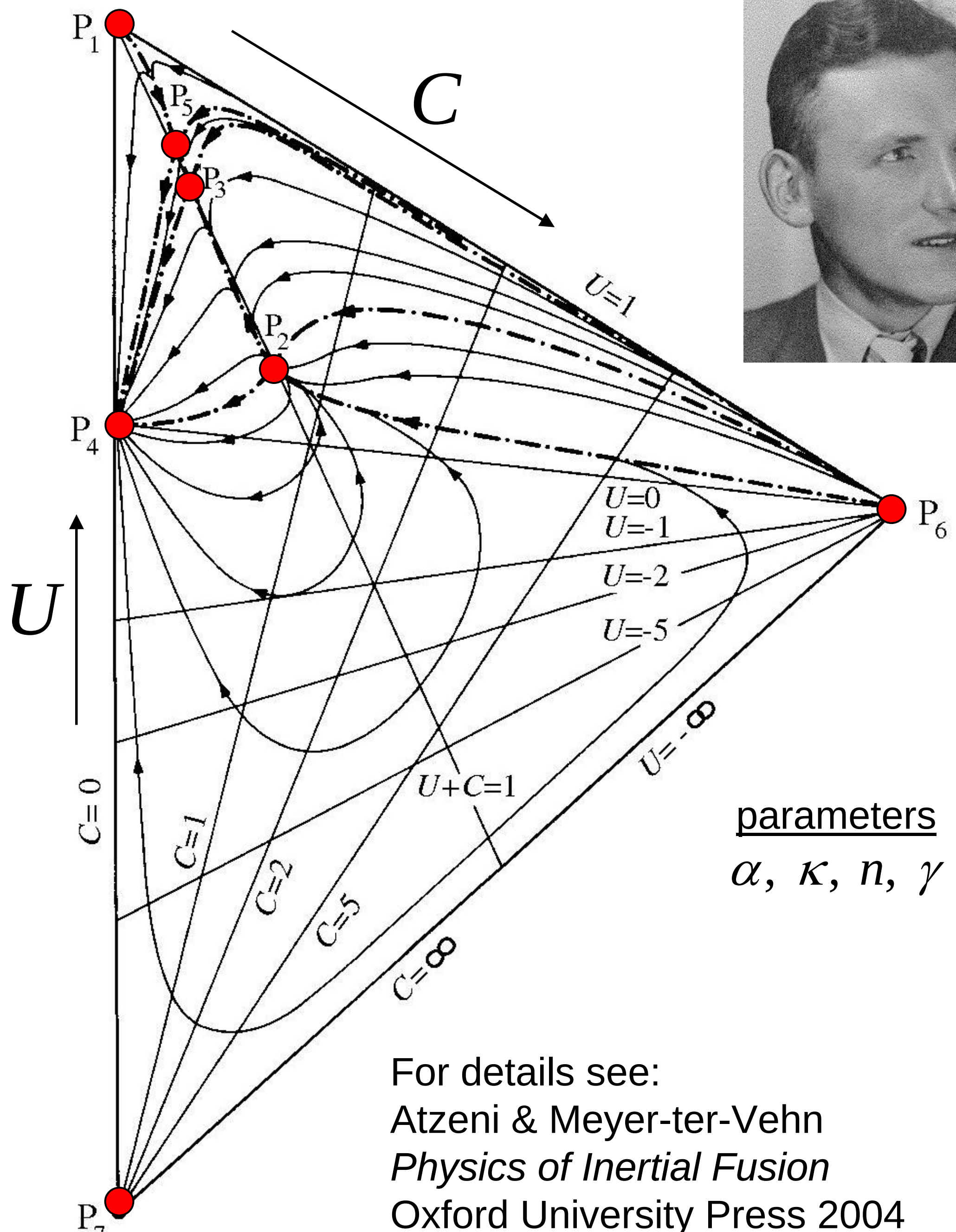
$$p / \rho^{\gamma} = A_0 \, r^{\varepsilon} \qquad (\varepsilon = \kappa(\gamma - 1) + 2\lambda)$$

isentropic $\varepsilon = 0$

Mach number

$$M = u_0 / c_0$$

Guderley solution



G. Guderley,
Luftfahrtforschung 19, 302 (1942)

$$\frac{dU}{dC} = \frac{\Delta_1(U,C)}{\Delta_2(U,C)}$$

solve to get $U(C)$

$$\frac{d\ln\xi}{dC} = \frac{\Delta(U,C)}{\Delta_2(U,C)}$$

integrate to get $\xi(C)$

$$C = C(\xi)$$

$$U = U(\xi)$$

$$G = G(U,C,\xi)$$

<u>parameters</u>

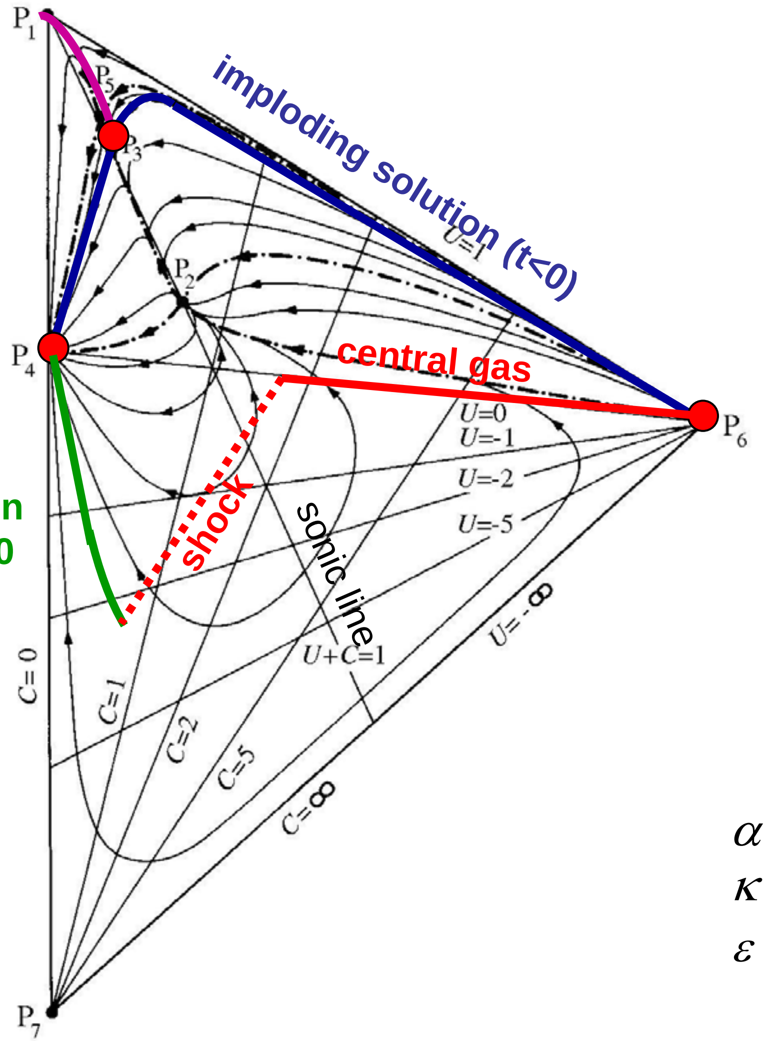
$\alpha,\ \kappa,\ n,\ \gamma$

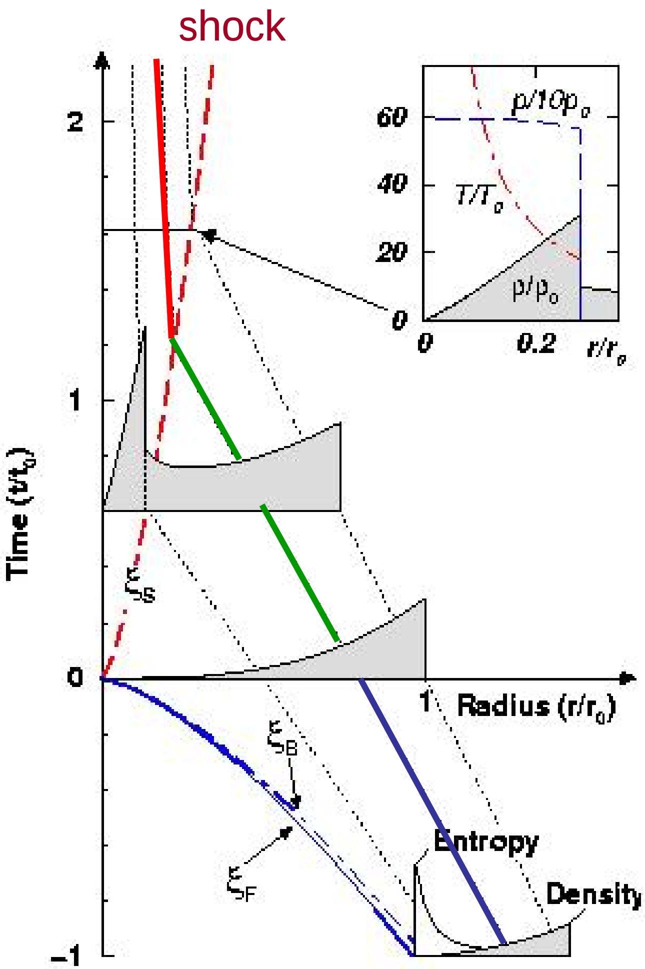
For details see:
Atzeni & Meyer-ter-Vehn
Physics of Inertial Fusion
Oxford University Press 2004

Self-similar implosion of hollow shell

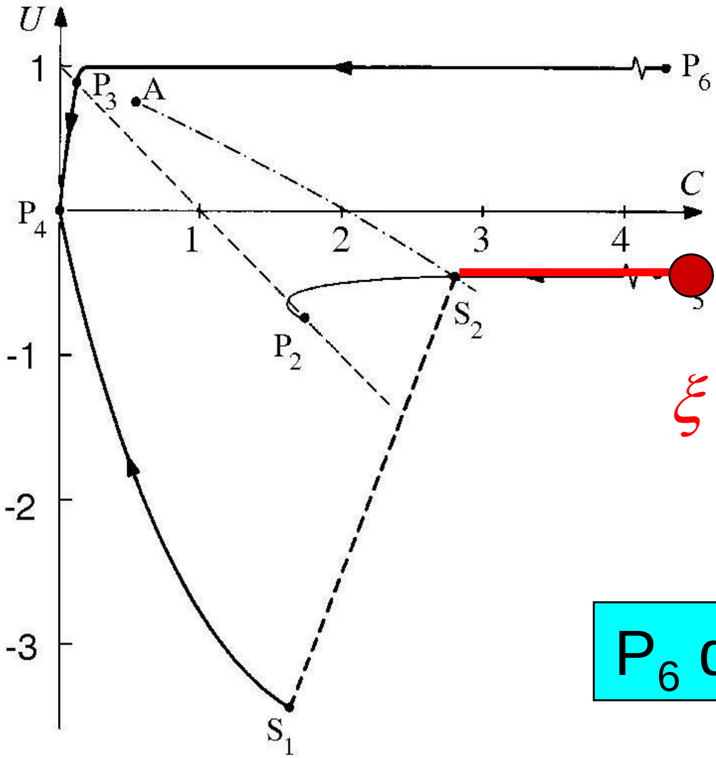
Meyer-ter-Vehn, Schalk, Z. Naturforschung 37a, 955 (1982)

Clark and Tabak, connected wrongly!





$\alpha = 0.7$
$\kappa = 3$
$\varepsilon = 20/7$



P_6: $C_6 = \infty, \qquad U_6 = \frac{2(1-1/\alpha)-\kappa}{n\gamma}$

$\xi = 0$

P_6 describes gas state in center

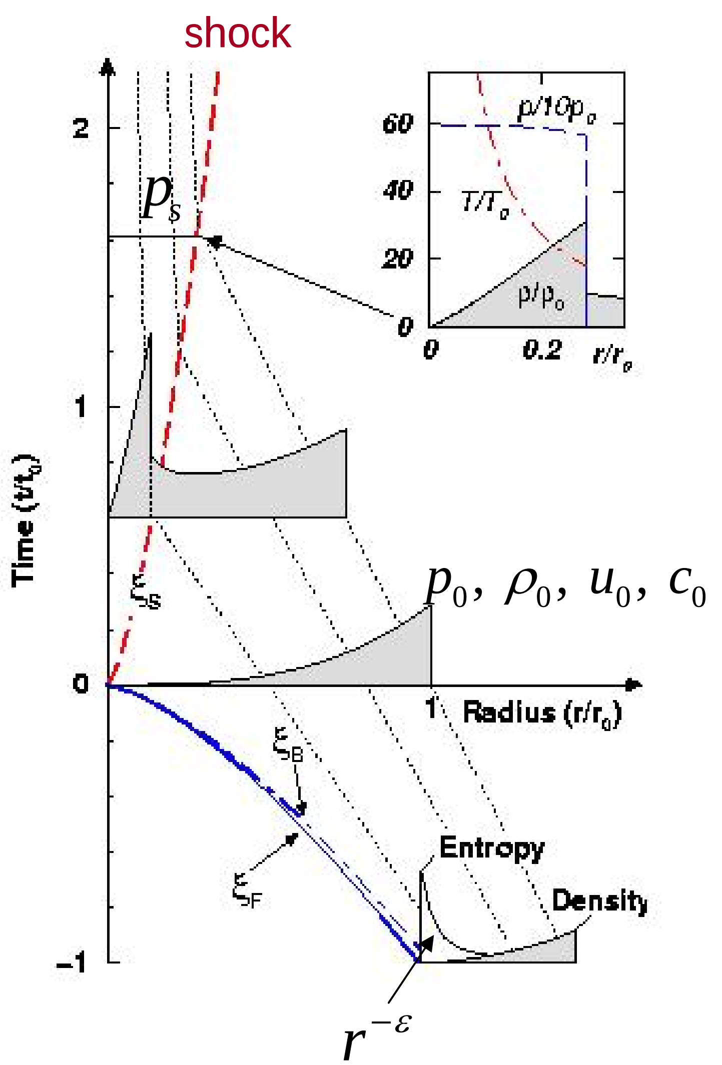
$$u(r,t) = \frac{2(1-1/\alpha)-\kappa}{n\gamma}) \ \alpha \frac{r}{t}$$

$$\rho(r,t) \ \sim \ r^{n\varepsilon/\nu} \ t^{\alpha(\kappa - n\varepsilon/\nu)}$$

$$T(r,t) \ \sim c^2 \sim \ r^{-n\varepsilon/\nu} \ t^{\alpha(-2\lambda + n\varepsilon/\nu)}$$

$$p(r,t) \ \sim \ r^0 \ t^{\alpha(\kappa - 2\lambda)} \qquad (\nu = n\gamma + \kappa - 2\lambda)$$

Mach number and entropy determine implosion



nearly uniform pressure behind shock p_s

at void closure (t=0)

$$M = u_0 / c_0 = \text{const}$$

key result

$$p_s / p_0 \cong 3.6\, M^3$$

$\alpha = 0.7$

$\kappa = 3$

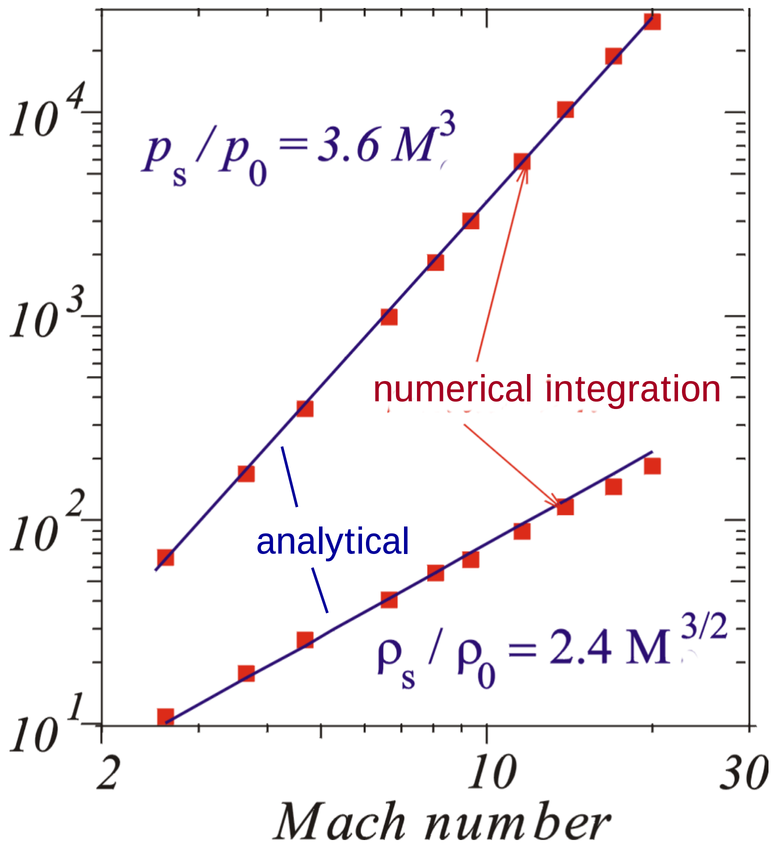
$\varepsilon = \kappa(\gamma - 1) + 2(1/\alpha - 1) = 2.85$

Stagnation pressure scaling

Meyer-ter-Vehn, Schalk, Z. Naturforschung 37a, 955 (1982)

Kemp, Meyer-ter-Vehn, Atzeni, PRL 86, 3336 (2001).

$$\frac{p_s}{p_0} \sim M^{\frac{2(n+1)}{\gamma+1}} \sim M^3 \quad \left\{ \begin{array}{l} \text{for} \\ n=3 \\ \gamma = 5/3 \end{array} \right.$$



<u>Mach number</u>

$$M = u_{imp} / c_0$$

$$c_0 \sim (p_0 / \rho_0)^{1/2}$$

$$p_0 \sim \alpha_{if} \rho_0^{5/3}$$

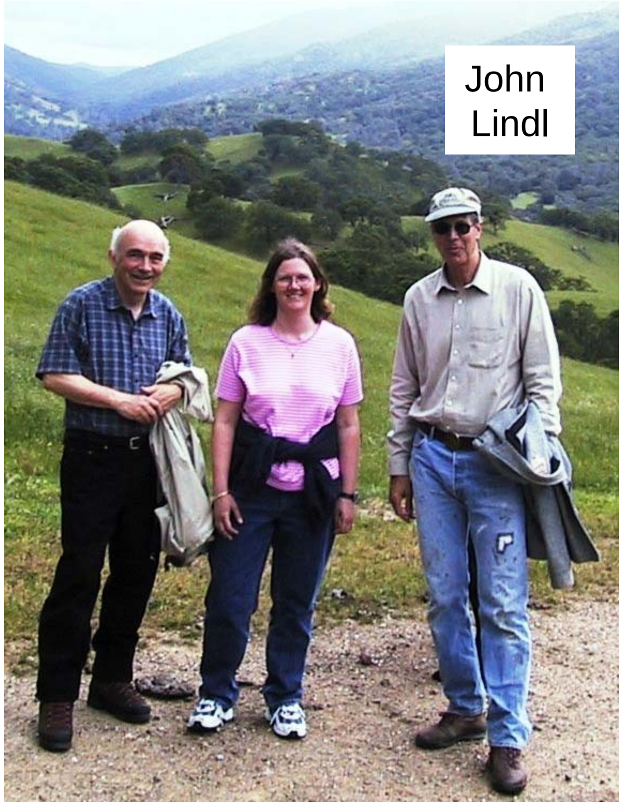
<u>Stagnation pressure</u>

$$p_s \sim u_{imp}^3 \ \alpha_{if}^{-9/10} p_a^{2/5}$$

Ignition energy scaling

$$E_f^{ign} \sim \frac{1}{p_s^2} \sim \frac{1}{p_a^2 M^6} \sim \alpha_{if}^{1.8}\, u_{imp}^{-6}\, p_a^{-0.8}$$

Atzeni, Meyer-ter-Vehn, Nucl.Fus. 41, 465 (2001)



$$E_f^{ign} \sim \alpha_{if}^{1.88\pm0.05}\, u_{imp}^{-5.89\pm0.12}\, p_a^{-0.77\pm0.12}$$

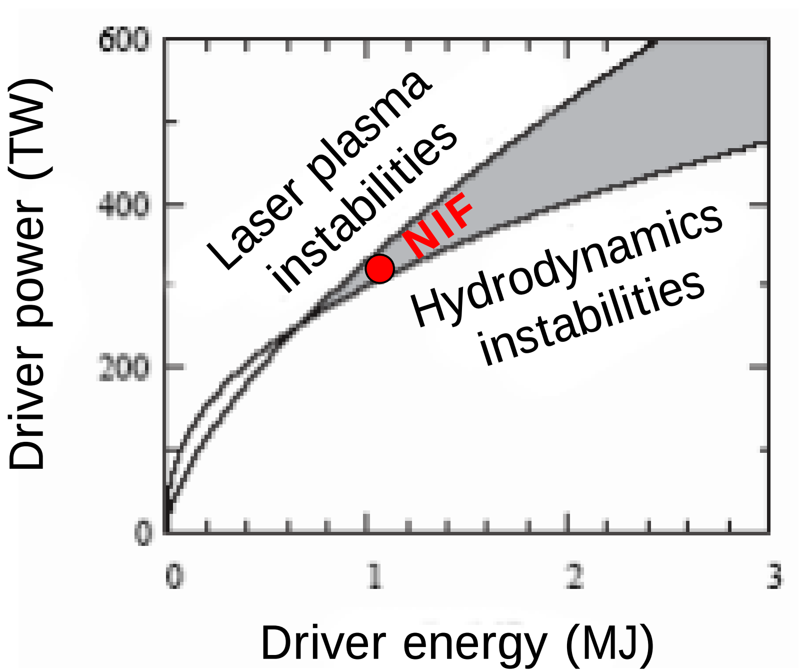
scaling extracted from large set of simulations

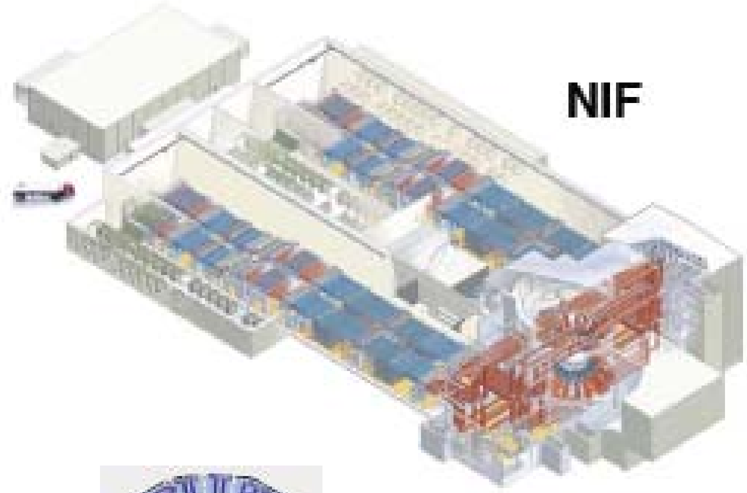
Herrmann, Tabak, Lindl, Nucl.Fus. 41, 99 (2001)

The ICF Ignition Window

J. Lindl et al. Physics of Plasmas 11, 339 (2004)

S. Atzeni & J. Meyer-ter-Vehn *Physics of Inertial Fusion* Oxford University Press (2004)

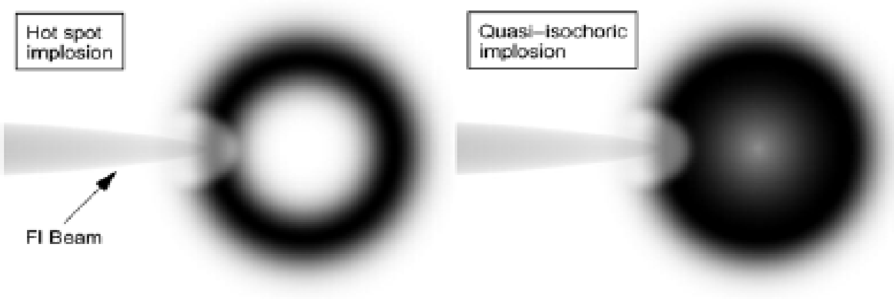




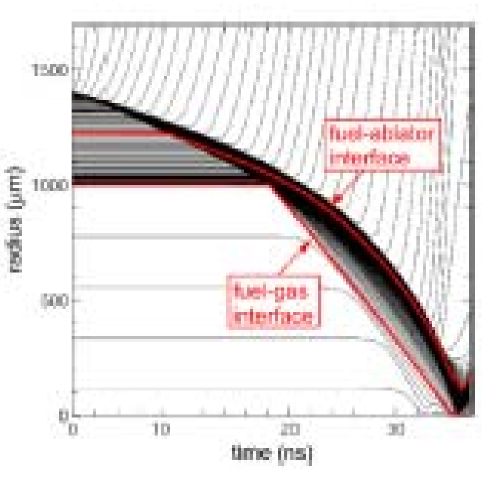
Nucl. Fusion 47 (2007) 1147–1156
doi:10.1088/0029-5515/47/9/011

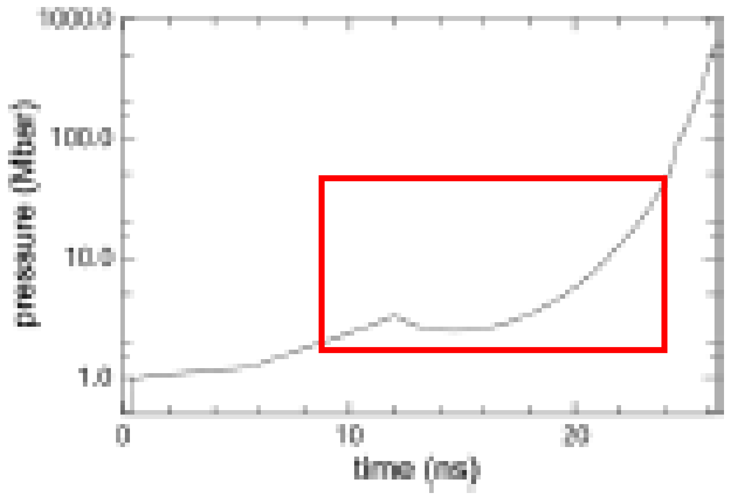
A self-similar isochoric implosion for fast ignition

D.S. Clark and M. Tabak



Look for solutions with $\varepsilon = 0$

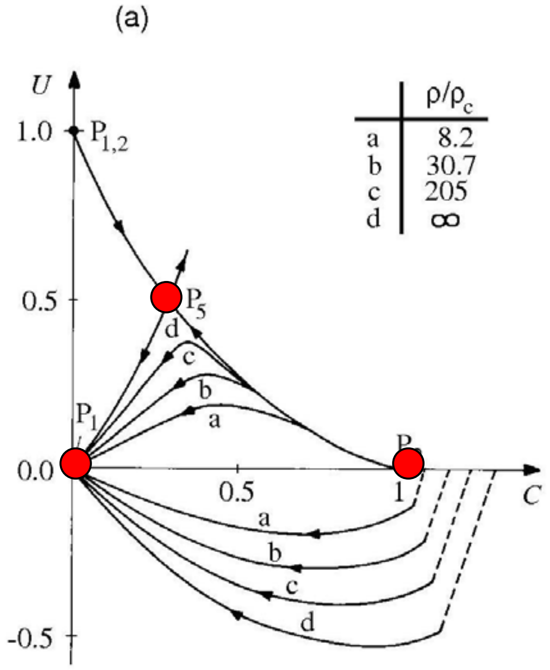


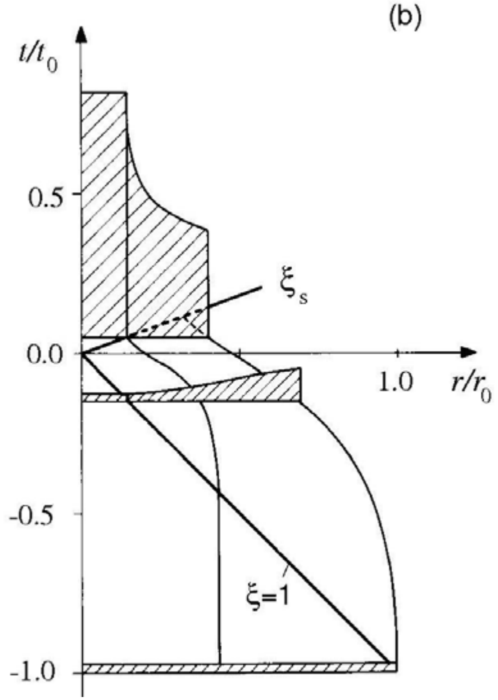


The ideal isochoric implosion :

Compressing a uniform gas at rest

into a uniform gas at rest of arbitrary density





<u>Disadvantage:</u>

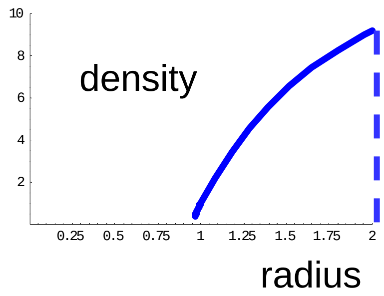
Low Mach number, return shock weak,

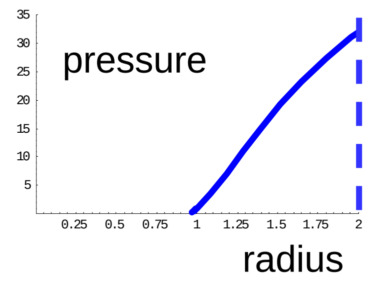
Almost full pressure of igniting fuel (>100 Gbar) has to be balanced by external piston

Book: Fig. 6.16 $n = 3, \ \gamma = 5/3, \ \alpha = 1, \ \kappa = 0, \ \varepsilon = 0$

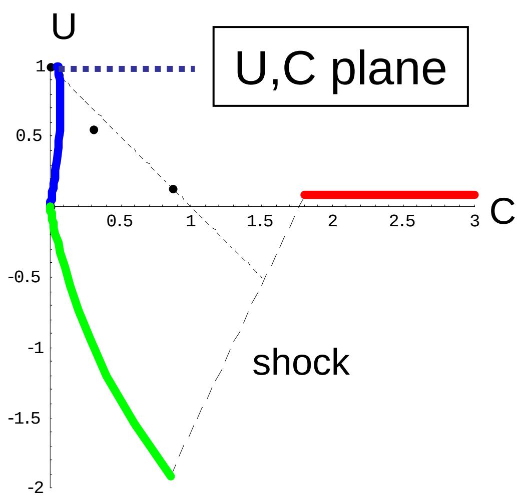
High Mach number, nearly isochoric solution: $\alpha = 0.9$, $\varepsilon = 0.1$

Imploding shell

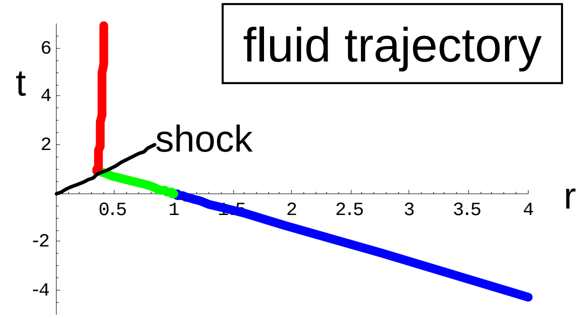




U,C plane

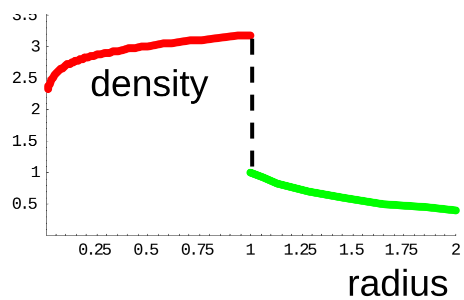


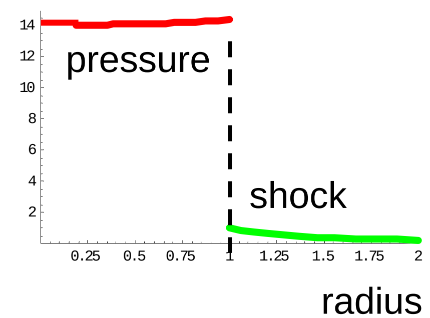
fluid trajectory



almost constant implosion velocity

Stagnating shell





Projection symmetry holds for $n = 2/(\gamma - 1)$

Book (Chapter 6.6)

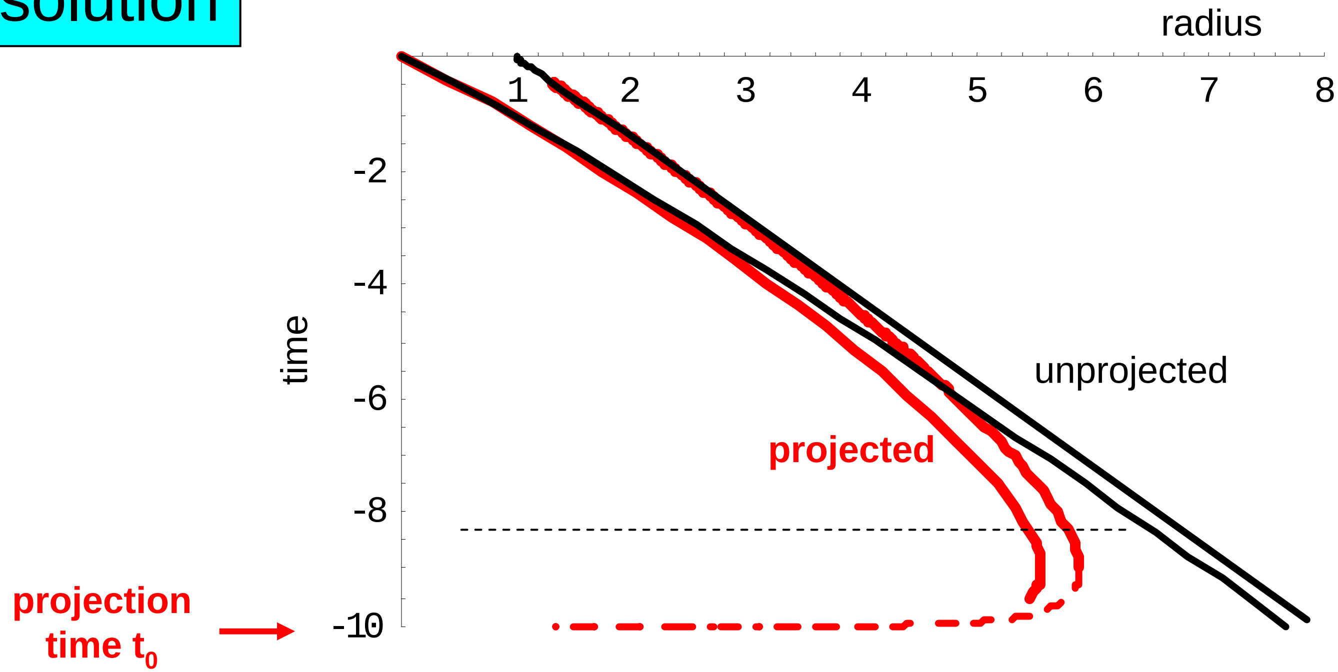
$$G_p = t^2\partial_t + rt\partial_r + (r - ut)\partial_u - ct\partial_c - n\rho t\partial_\rho$$

allows to generate new solution from known solution

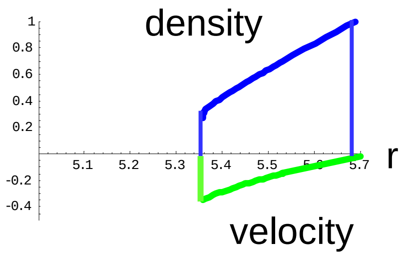
$$\exp(\varepsilon G_p)\{r,t,u,c,\rho\}\left\{r^{(0)},t^{(0)},u^{(0)},c^{(0)},\rho^{(0)}\right\} = \left\{r^{(\varepsilon)},t^{(\varepsilon)},u^{(\varepsilon)},c^{(\varepsilon)},\rho^{(\varepsilon)}\right\}$$

old solution	projected new solution
$\xi = \dfrac{r}{\lvert t\rvert^{\alpha}}$	$\xi = \dfrac{r}{\lvert t\rvert^{\alpha}(1+t/t_0)^{1-\alpha}}$
$u(r,t) = \alpha\dfrac{r}{t}U(\xi)$	$u(r,t) = \dfrac{r}{t(1+t/t_0)}\left[\alpha U(\xi) + t/t_0\right]$
$c(r,t) = \alpha\dfrac{r}{t}C(\xi)$	$c(r,t) = \dfrac{r}{t(1+t/t_0)}\,\alpha C(\xi)$
$\rho(r,t) = \rho_0 r^{\kappa} G(\xi)$	$\rho(r,t) = \rho_0\dfrac{r^{\kappa}}{t(1+t/t_0)^{n+\kappa}}\,G(\xi)$

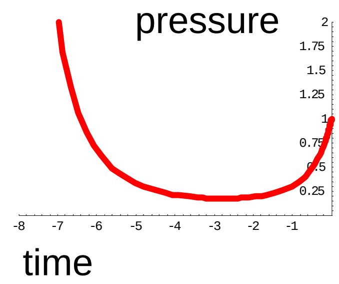
Projected solution

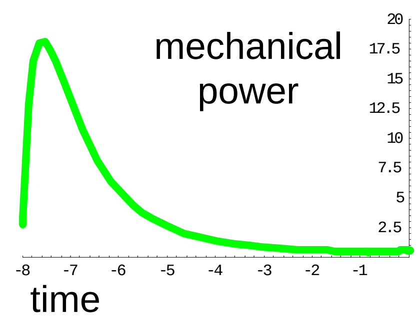


profiles at t = - 8



applied to outer trajectory:





Conclusions

Similarity solutions provide deep insight into the gas dynamics of ICF implosions

They also serve as a guide for isochoric implosions in the context of fast ignition